# Notes 12/12 – All about $i$ – imaginary numbers

<u>Definition</u>: $i = \sqrt{-1}$ OR $i^2 = -1$

→ Now we <u>can</u> take even ($\sqrt{\ }$, $\sqrt[4]{\ }$) root of a negative number, but the answer is <u>imaginary</u>.

# Square roots of negative numbers

→ need $i$ to $\sqrt{\ }$ a negative!

Ex 1: $\sqrt{-9} = \sqrt{9}\sqrt{-1} = 3i$

since $-9 = 9 \cdot (-1)$

simplified each $\sqrt{\ }$

Ex 2: $\sqrt{-25} = \sqrt{25}\sqrt{-1} = \boxed{5i}$

Ex 3: $\sqrt{-4x^2} = \sqrt{4}\sqrt{-1}\sqrt{x^2} = 2xi$

Do:

1) $\sqrt{-49}$

2) $\sqrt{-144}$

3) $\sqrt{121}$

4) $\sqrt{-36y^2}$

# Adding and Subtracting Complex Numbers

a <u>complex number</u> : $a + bi$

real component (no $i$) — $a$; imaginary component ($i$) — $bi$

Ex: $3+2i$, $8-7i$

- To add or subtract complex numbers, just treat $i$ like a variable and combine like terms!

<u>Ex 1</u>: $(3+2i) + (5+6i) = \boxed{\underset{3+5}{8} + \underset{2i+6i}{8i}}$

<u>Ex 2</u>: $(5+3i) - (2-4i)$

$= 5 + 3i - 2 + 4i = \boxed{3+7i}$

<u>Do</u>: 1) $(5+6i) + (9+3i)$

2) $(7+4i) + (3-5i)$

3) $(6-2i) - (4+9i)$

4) $(-2+3i) - (6-8i)$

# Multiplying Imaginary Numbers

- Do the first steps like normal
- When you get $i^2$, simplify $i^2 = -1$

<u>Ex 1</u>: $(6i)(2i) = 12i^2 = 12(-1) = -12$

since $i^2 = -1$

<u>Ex 2</u>: $(-4i)(3i) = -12i^2 = -12(-1) = 12$

<u>Ex 3</u>: $f(x) = 2x^2 + 1$, find $f(3i)$

$$\begin{aligned} f(3i) &= 2(3i)^2 + 1 \\ &= 2(9i^2) + 1 \\ &= 18i^2 + 1 \\ &= 18(-1) + 1 = -18 + 1 = -17 \end{aligned}$$

<u>Do</u>:

1) $(-4i)(5i)$

2) $(10i)(3i)$

3) $f(x) = 3x^2 - 4$, find $f(2i)$

# Multiplying Complex #5

- Do all of the first steps like normal!
- At the end, make $i^2 = -1$ and keep simplifying

Ex 1: $(2+4i)(3-i)$

$= 6 - 2i + 12i - 4i^2$

$= 6 + 10i - 4i^2$

$= 6 + 10i - 4(-1)$ since $i^2 = -1$

$= 6 + 10i + 4$

$= 10 + 10i$

Ex 2: $(-4+5i)(-4-5i)$

$= 16 + 20i - 20i - 25i^2$

$= 16 - 25i^2$

$= 16 - 25(-1)$

$= 16 + 25$

$= 41$

Do: $(6+4i)(9-3i)$

2) $(2-6i)(2+6i)$